# Urea-Formaldehyde Reaction System. An Experimental Investigation

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#### **Synopsis**

Batch production of urea-formaldehyde resins at temperatures up to 100°C takes several hours for completion. Reduction of the batch time may be possible with the higher reaction rates obtained at higher temperatures and increased pressures. In order to investigate this possibility, an experimental technique to obtain the necessary kinetic data, without loss of formaldehyde by evaporation, was developed. The results are compared with earlier low-temperature data extrapolated to the present range of interest. The results were interpreted on the basis of the successive reaction of two or three molecules of formaldehyde with a molecule of urea.

#### **Rate Equations**

In order to carry out the chemical engineering design procedure for a reactor producing UF (urea-formaldehyde) resins, appropriate rate equations would be required of the form

$$r = \pm \frac{dc}{dt} = f[k(T), \text{ concentrations}]$$
 (1)

where r = rate of appearance or disappearance of a chemical, c = concentration of that chemical, and t = time elapsed from start of reaction.

It has been established<sup>1</sup> that the combination of urea and formaldehyde begins with a series of addition reactions<sup>2</sup> followed by condensation reactions<sup>3</sup>; that the speed and extent of reaction are dependent on temperature, pH, and U:F ratio, although the reaction rate is essentially constant in the pH range 4–9 at constant temperature<sup>4</sup> and that UF<sub>3</sub> is produced in significant quantities only at low U:F ratios.<sup>5</sup> Therefore, since the commercial process usually involves U:F molar ratios between 1:1.33 and 1:2.2, within the pH range 4–9, it is reasonable to assume initially that the reactions taking place are

$$\mathbf{U} + \mathbf{F} \underset{k_2}{\overset{k_1}{\longleftrightarrow}} \mathbf{U} \mathbf{F}_1 \tag{2}$$

$$UF_1 + F \underset{k_4}{\overset{k_3}{\longleftrightarrow}} UF_2 \tag{3}$$

so that the rate equations become

$$r_F = \frac{-d[F]}{dt} = k_1[U][F] - k_2[UF_1] + k_3[F][UF_1] - k_4[UF_2]$$
(4)

$$r_U = \frac{-d[U]}{dt} = k_1[U][F] - k_2[UF_1]$$
(5)

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where [ ] denotes concentration in moles per unit volume. A mass balance on formaldehyde and urea gives

$$[\mathbf{F}] + [\mathbf{F}]_{\mathbf{UF}_1} + [\mathbf{F}]_{\mathbf{UF}_2} = [\mathbf{F}_0] = [\mathbf{F}] + [\mathbf{UF}_1] + 2[\mathbf{UF}_2]$$
(6)

$$[U] + [U]_{UF_1} + [U]_{UF_2} = [U_0] = [U] + [UF_1] + [UF_2]$$
(7)

Solving eqs. (6) and (7) in terms of  $UF_1$  and  $UF_2$  gives

$$[UF_1] = 2[U_0] - 2[U] - [F_0] + [F]$$
(8)

$$[UF_2] = [U] - [F] - [U_0] + [F_0]$$
(9)

Substituting eqs. (8) and (9) into (4) and (5) yields

$$r_{F} = \frac{-d[F]}{dt} = k_{1}[U][F] - k_{2}(4[U_{0}] - 2[U] - 2[F_{0}] + 2[F]) + k_{3}[F](2[U_{0}] - 2[U] - [F_{0}] + [F] - k_{4}[U] - [F] - [U_{0}] + [F_{0}])$$
(10)  
$$-d[U] + [U][F] - k_{3}(U][F] - k_{4}[U] - [F] - [U_{0}] + [F_{0}])$$
(11)

$$r_U = \frac{-d[U]}{dt} = k_1[U][F] - k_2 \left(4[U_0] - 4[U] - 2[F_0] + 2[F]\right)$$
(11)

At a particular temperature, solution of the second-order nonlinear simultaneous differential eqs. (10) and (11) is possible using a computer, providing that values of the rate constants  $k_1$  to  $k_4$  are available. These were obtained by evaluating the data of de Jong and de Jonge<sup>6,7</sup> plotted as  $\ln k$  vs. 1/T, using the Arrhenius equation

$$k = A \, \exp\!\left(\frac{-\Delta E}{RT}\right)$$

or

$$\ln k = \ln A - \left(\frac{\Delta E}{R} \cdot \frac{1}{T}\right) \tag{12}$$

Errors in estimating the intercept,  $\ln A$ , were reduced by calculating the slopes of the curves from a knowledge of the activation energy  $\Delta E$ . The following values were obtained:

$$k_1^0 = 10^{5.81} \exp\left(-\frac{6542.5}{T}\right)$$
l/mole sec (13)

$$k_2^0 = 10^{6.31} \exp\left(-\frac{9562.15}{T}\right) 1/\text{sec}$$
 (14)

$$k_3^0 = 10^{4.26} \exp\left(-\frac{7045.8}{T}\right)$$
l./mole sec (15)

$$k_4^0 = 10^{7.15} \exp\left(-\frac{9562.15}{T}\right) 1/\text{sec}$$
 (16)

where the superscript 0 indicates that the rate constant originated from the data of de Jong and de Jonge.

Values for [U] and [F] were obtained<sup>1</sup> by integrating eqs. (10) and (11) and inserting eqs. (13)–(16) on a Honeywell 316 computer using the package Aston Simulation Program<sup>8</sup> and the Runga-Kutta fourth-order routine.<sup>9</sup> A flow sheet for the program is shown in Figure 1.

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## EXPERIMENTAL RESULTS AND DISCUSSION

The data of de Jong and de Jonge used in the kinetic model were obtained under conditions where the condensation reactions were minimized. Since this is unlikely to be the case in the industrial production situation, experimental results were obtained to test the model.<sup>1</sup> Commercial 36% formalin and urea were charged to a glass reactor fitted with stirrer and condenser. The pH was adjusted and the temperature controlled in the range of 20–80°C. Samples taken from the reactor at selected intervals were analyzed for free formaldehyde by the acidimetric sulfite method. The initial U:F ratio in the charge was always

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1:1.33. Results are listed in Table I and shown in Figure 2. Comparison of predicted and experimental results at 25 and 80°C in Figure 3 is very good at low temperatures, but divergence occurs at higher temperatures.

The discrepancies noted at higher temperatures could arise from the following factors:

(1) inaccuracy caused by the loss of the more volatile component, formaldehyde, from the reactor either in the form of a gas or a solid paraformaldehyde deposited on cold surfaces,

(2) the effect of the unavoidable reaction occurring while the reactor contents are being heated to the selected temperature,

(3) the reaction scheme suggested in eqs. (2) and (3) may not be adequate at higher temperatures because of further reactions of the addition or condensation type.

It was apparent that extrapolation of the model to temperatures higher than 60°C would be unwise and that an experimental method capable of avoiding items (1) and (2) above would be advantageous.

| Experimental Results in Open Reactor* |           |        |  |  |  |  |  |
|---------------------------------------|-----------|--------|--|--|--|--|--|
| Temp, °C                              | Time, min | [F], % |  |  |  |  |  |
|                                       | 0         | 23.40  |  |  |  |  |  |
|                                       | 60        | 13.10  |  |  |  |  |  |
| 25                                    | 120       | 9.90   |  |  |  |  |  |
|                                       | 240       | 6.60   |  |  |  |  |  |
|                                       | 360       | 4.75   |  |  |  |  |  |
|                                       | 0         | 23.40  |  |  |  |  |  |
|                                       | 15        | 14.50  |  |  |  |  |  |
|                                       | 30        | 11.30  |  |  |  |  |  |
| 40                                    | 60        | 7.60   |  |  |  |  |  |
|                                       | 120       | 4.70   |  |  |  |  |  |
|                                       | 240       | 2.25   |  |  |  |  |  |
|                                       | 360       | 1.75   |  |  |  |  |  |
|                                       | 0         | 23.40  |  |  |  |  |  |
|                                       | 10        | 8.80   |  |  |  |  |  |
|                                       | 20        | 5.40   |  |  |  |  |  |
|                                       | 30        | 3.75   |  |  |  |  |  |
| 60                                    | 60        | 1.90   |  |  |  |  |  |
| ·                                     | 120       | 1.00   |  |  |  |  |  |
|                                       | 240       | 0.50   |  |  |  |  |  |
|                                       | 360       | 0.25   |  |  |  |  |  |
|                                       | 0         | 23.40  |  |  |  |  |  |
|                                       | 5         | 7.40   |  |  |  |  |  |
|                                       | 10        | 3.75   |  |  |  |  |  |
|                                       | 20        | 1.70   |  |  |  |  |  |
| 80                                    | 30        | 1.00   |  |  |  |  |  |
|                                       | 60        | 0.70   |  |  |  |  |  |
|                                       | 120       | 0.60   |  |  |  |  |  |
|                                       | 240       | 0.50   |  |  |  |  |  |
|                                       | 360       | 0.50   |  |  |  |  |  |

TABLE I erimental Results in Open Reacto

<sup>a</sup> U:F Ratio 1:1.33.



Fig. 2. Concentration-time curves for formaldehyde. Initial pH 8, U:F 1:1.33.

## **Modified Experimental Procedure for UF Reactions**

The method used by Gordunov<sup>10</sup> was modified to avoid the problems in timing the reaction and controlling the temperature of the reaction. The static sealed tube was replaced by a Y-shaped reactor made of glass (Fig. 4). Its size was chosen to ensure that an adequate sample was available for formaldehyde analysis with minimum vapor space. The latter allowed minimum vaporization to pressurize the system, thus avoiding loss of formaldehyde.

A predetermined quantity of prilled urea was charged to one arm of the reactor. Formalin at a chosen pH was carefully charged to the other arm using a hypodermic syringe to avoid contact with the urea. The charging arm was then sealed in a flame in such a way that the pressure generated on heating the contents would not break the reactor. It was then held in a laboratory clamp which also acted as the shaft to a suitable electric stirrer motor. This was carefully posi-



Fig. 3. Prediction of low-temperature UF kinetics: ( $\triangle$ ) experimental data; ( $\odot$ ) predicted data.

tioned so that the reactants were well immersed in a constant-temperature bath without contacting one another. After a sufficient period of time for the reactants to reach the required temperature, the motor was switched on to mix the reactants together, with the reactor now behaving as a "Y-cone blender." At the end of the required period, the reactor was removed and broken immediately into a quantity of ice water to quench the reaction. (The ice water also served as part of the water required in the analytical procedure.)

Experiments were carried out at U:F molar ratios of 1:1.33 and 1:2.2 over the temperature range of 25–160°C with the pH in the band 4–9. Results for formaldehyde<sup>11</sup> are shown in Table II and representative values are shown in Figure 5. Comparison with Table I indicates that the present method produces slightly higher formaldehyde concentrations than the conventional open reactor, suggesting that loss of formaldehyde from the system has been prevented.



Fig. 4. Glass reactor.

## **Modeling of Experimental Results**

Since eqs. (2) and (3) proved to be inadequate over the entire temperature range examined, the third addition reaction was included in the mechanism:

$$UF_2 + F \underset{k_6}{\overset{k_5}{\longleftrightarrow}} UF_3 \tag{17}$$

The rate equations then become

$$r_{F} = \frac{-d[F]}{dt} = k_{1} [U][F] - k_{2} [UF_{1}] + k_{3}[F][UF_{1}] - k_{4}[UF_{2}] + k_{5}[F][UF_{2}] - k_{6}[UF_{3}]$$
(18)

$$r_U = \frac{-d[U]}{dt} = k_1[U][F] - k_2[UF_1]$$
(19)

$$r_{UF_1} = \frac{d[\mathrm{UF}_1]}{dt} = k_2[\mathrm{UF}_1] - k_1[\mathrm{U}][\mathrm{F}] + k_3[\mathrm{F}][\mathrm{UF}_1] - k_4[\mathrm{UF}_2]$$
(20)

and the mass balances become

$$[U] + [UF_1] + [UF_2] + [UF_3] = [U_0]$$
(21)

$$[\mathbf{F}] + [\mathbf{UF}_1] + 2[\mathbf{UF}_2] + 3[\mathbf{UF}_3] = [\mathbf{F}_0]$$
(22)

so that

$$[UF_2] = 3([U_0] - [U] + [F] - [F_0] - 2[UF_1])$$
(23)

$$[\mathbf{UF}_3] = 2([\mathbf{U}] - [\mathbf{U}_0]) + [\mathbf{F}_0] - [\mathbf{F}] + [\mathbf{UF}_1]$$
(24)

Substitution of eqs. (23) and (24) into (18), (19), and (20) provides the necessary rate equations for U, F, and UF<sub>1</sub> and can be treated as before if values of the rate constants in eq. (17) are available. Lack of data prevents  $k_5$  and  $k_6$  being calculated in the same manner as eqs. (13)–(16), but de Jong and de Jonge<sup>12</sup> quote approximate values for  $k_5$  and the equilibrium constant for reaction (17) at 35°C. Using these values gives the following expressions for  $k_5$  and  $k_6$ :

| Experimental Predicted |              |               |           |         |                   |         |         |         |             |         |
|------------------------|--------------|---------------|-----------|---------|-------------------|---------|---------|---------|-------------|---------|
| Temp.,                 | Time,        | [F]           | [F],      | [F],    | $\underline{k_1}$ | $k_2$   | $k_3$   | $k_4$   | $k_5$       | $k_6$   |
| °C                     | min          | %             | mole/l.   | mole/l. | $k_1^0$           | $k_2^0$ | $k_3^0$ | $k_4^0$ | $k_{5}^{0}$ | $k_6^0$ |
|                        |              |               |           | II.     | F Patio 1         | •1.99   |         |         |             |         |
| 25                     | 0.0          | <b>93 39</b>  | 0 1 867   | 0.1867  | 0.8534            | 1 0034  | 0 1034  | 1 0144  |             |         |
| 20                     | 10.0         | 20.02         | 7 9500    | 7 0500  | 0.0004            | 1.0034  | 0.1034  | 1.0144  |             |         |
|                        | 20.0         | 17 00         | 7.0500    | 7.0696  |                   |         |         |         |             |         |
|                        | 20.0         | 16 19         | 6 9750    | C 2901  |                   |         |         |         |             |         |
|                        | 40.0         | 14.05         | 5 9975    | 5 9497  |                   |         |         |         |             |         |
|                        | 40.0         | 14.90         | 5.0075    | 5 4114  |                   |         |         |         |             |         |
|                        | 00.0<br>60.0 | 10.90         | 5 1059    | 5.4114  |                   |         |         |         |             |         |
|                        | 190.0        | 12.90         | 2 0 4 0 0 | 0.0001  |                   |         |         |         |             |         |
|                        | 240.0        | 10.04<br>6 75 | 0.9490    | _       |                   |         |         |         |             |         |
|                        | 240.0        | 0.70<br>E 70  | 2.0049    | _       |                   |         |         |         |             |         |
|                        | 300.0        | 0.70          | 2.2100    |         |                   |         |         |         |             |         |
| 40                     | 0.0          | 00.00         | 0 1 867   | 0 1967  | 0 0030            | 1.0034  | 1.0094  | 1 0144  |             |         |
| 40                     | 10.0         | 16 19         | 6 2750    | 6 9454  | 0.3030            | 1.0004  | 1.0034  | 1.0144  | _           |         |
|                        | 20.0         | 10.10         | 5.0250    | 1 0033  |                   |         |         |         |             |         |
|                        | 20.0         | 10.85         | 4 9750    | 4.5500  |                   |         |         |         |             |         |
|                        | 40.0         | 10.00         | 2 7500    | 4.2000  |                   |         |         |         |             |         |
|                        | 50.0         | 9.02          | 2 2750    | 2 2200  |                   |         |         |         |             |         |
|                        | 60.0         | 0.07          | 2.0100    | 3.3200  |                   |         |         |         |             |         |
|                        | 190.0        | 1.01          | 0 1 2 5 7 | 3.0494  |                   |         |         |         |             |         |
|                        | 120.0        | 0.40          | 1 2649    |         |                   |         |         |         |             |         |
|                        | 240.0        | 0.47          | 0.0626    | _       |                   |         |         |         |             |         |
|                        | 240.0        | 1.62          | 0.5050    |         |                   |         |         |         |             |         |
|                        | 300.0        | 1.00          | 0.0411    | _       |                   |         |         |         |             |         |
| 60                     | 0.0          | 23.32         | 9 1867    | 9 1867  | 1 0068            | 1 0068  | 4 0068  | 1 0289  | _           |         |
| 00                     | 10.0         | 8 75          | 3 4415    | 3 4306  | 1.0000            | 1.0000  | 4.0000  | 1.0200  |             |         |
|                        | 20.0         | 5 45          | 2 1 4 8 9 | 2 1930  |                   |         |         |         |             |         |
|                        | 30.0         | 4 50          | 1 7699    | 1 6078  |                   |         |         |         |             |         |
|                        | 40.0         | 3.94          | 1.7055    | 1.0010  |                   |         |         |         |             |         |
|                        | 50.0         | 2.66          | 1.0500    | 1.0090  |                   |         |         |         |             |         |
|                        | 60.0         | 2.00          | 1.0000    | 0.8307  |                   |         |         |         |             |         |
|                        | 90.0         | 1 49          | 0.5585    |         |                   |         |         |         |             |         |
|                        | 120.0        | 1.42          | 0.4759    |         |                   |         |         |         |             |         |
|                        | 180.0        | 0.58          | 0.2281    |         |                   |         |         |         |             |         |
|                        | 100.0        | 0.00          | 0.2201    |         |                   |         |         |         |             |         |
| 80                     | 0.0          | 23.32         | 9.1867    | 9.1867  | 1.0136            | 1.0136  | 5.0136  | 5.0578  | _           |         |
| 00                     | 5.0          | 6.37          | 2,5093    | 2.4630  | 1.0100            | 1.0100  | 0.0100  | 0.0010  |             |         |
|                        | 10.0         | 3.58          | 1.4109    | 1.3902  |                   |         |         |         |             |         |
|                        | 15.0         | 2.51          | 0.9900    | 0.9305  |                   |         |         |         |             |         |
|                        | 20.0         | 1.88          | 0.7394    | 0.6821  |                   |         |         |         |             |         |
|                        | 25.0         | 1.52          | 0.6000    | 0.5341  |                   |         |         |         |             |         |
|                        | 30.0         | 1.36          | 0.5349    | 0.4413  |                   |         |         |         |             |         |
|                        | 60.0         | 0.98          | 0.3854    | _       |                   |         |         |         |             |         |
|                        | 120.0        | 0.86          | 0.3382    |         |                   |         |         |         |             |         |
|                        |              |               |           |         |                   |         |         |         |             |         |
| 120                    | 0.0          | 23.32         | 9.1867    | 9.1867  | 1.0000            | 1.0000  | 5.0000  | 1.0000  | 0.8333      | 1.2000  |
|                        | 1.17         | 2.96          | 1.1642    |         |                   |         |         |         |             |         |
|                        | 2.5          | 1.49          | 0.5860    | 0.6138  |                   |         |         |         |             |         |
|                        | 5.0          | 0.55          | 0.2160    | 0.2011  |                   |         |         |         |             |         |
|                        | 7.5          | 0.34          | 0.1350    | 0.1332  |                   |         |         |         |             |         |
|                        | 10.0         | 0.33          | 0.1284    | 0.1208  |                   |         |         |         |             |         |
|                        | 12.5         | 0.30          | 0.1195    | 0.1185  |                   |         |         |         |             |         |
|                        | 15.0         | 0.27          | 0.1048    | 0.1176  |                   |         |         |         |             |         |

TABLE II Experimental and Predicted Results

|        |       | Experimental |         | Predicted |                    |                    |                    |                    |                    |                    |
|--------|-------|--------------|---------|-----------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Temp., | Time, | [F]          | [F],    | [F],      | $-k_1$             | $k_2$              | $k_3$              | $k_4$              | $k_5$              | $k_6$              |
| °C     | min   | %            | mole/l. | mole/l.   | $\overline{k_1^0}$ | $\overline{k_2^0}$ | $\overline{k_3^0}$ | $\overline{k_4^0}$ | $\overline{k_5^0}$ | $\overline{k_6^0}$ |
|        |       |              |         |           |                    |                    |                    |                    |                    |                    |
|        | 20.0  | 0.32         | 0.1259  | —         |                    |                    |                    |                    |                    |                    |
|        | 30.0  | 0.15         | 0.0590  | _         |                    |                    |                    |                    |                    |                    |
|        |       |              |         |           |                    |                    |                    |                    |                    |                    |
| 160    | 0.0   | 23.32        | 9.1867  | 9.1867    | 1.0000             | 1.0000             | 8.0000             | 4.0000             | 0.8333             | 1.2000             |
|        | 1.0   | 0.95         | 0.3736  | 0.3761    |                    |                    |                    |                    |                    |                    |
|        | 2.0   | 0.90         | 0.3562  | 0.3423    |                    |                    |                    |                    |                    |                    |
|        | 3.0   | 0.86         | 0.3375  | 0.3330    |                    |                    |                    |                    |                    |                    |
|        | 4.0   | 0.85         | 0.3337  | 0.3292    |                    |                    |                    |                    |                    |                    |
|        | 5.0   | 0.84         | 0.3325  | 0.3275    |                    |                    |                    |                    |                    |                    |
|        | 6.0   | 0.83         | 0.3287  | 0.3268    |                    |                    |                    |                    |                    |                    |
|        | 10.0  | 0.72         | 0.2832  | _         |                    |                    |                    |                    |                    |                    |
|        | 20.0  | 0.62         | 0.2438  | _         |                    |                    |                    |                    |                    |                    |
|        | 30.0  | 0.62         | 0.2438  | _         |                    |                    |                    |                    |                    |                    |
|        |       |              |         | U:        | F Ratio            | 1:2.2              |                    |                    |                    |                    |
| 40     | 0.0   | 27.12        | 10.4970 | 10.4970   | 1.0000             | 1.0000             | 1.0000             | 1.0000             | 0.8333             | 1.2000             |
|        | 5.0   | 21.96        | 8.4998  |           |                    |                    |                    |                    |                    |                    |
|        | 10.0  | 21.10        | 8.1669  | 8.0954    |                    |                    |                    |                    |                    |                    |
|        | 20.0  | 18.29        | 7.0793  | 7.0166    |                    |                    |                    |                    |                    |                    |
|        | 30.0  | 16.86        | 6.5250  | 6.4295    |                    |                    |                    |                    |                    |                    |
|        | 40.0  | 15.70        | 6.0750  | 6.0714    |                    |                    |                    |                    |                    |                    |
|        | 50.0  | 14.92        | 5.7750  | 5.8345    |                    |                    |                    |                    |                    |                    |
|        | 60.0  | 14.10        | 5.4575  | 5.6665    |                    |                    |                    |                    |                    |                    |
|        | 90.0  | 11.35        | 4.3931  | _         |                    |                    |                    |                    |                    |                    |
|        | 120.0 | 10.95        | 4.2383  | _         |                    |                    |                    |                    |                    |                    |
|        |       |              |         |           |                    |                    |                    |                    |                    |                    |
| 60     | 0.0   | 27.12        | 10.4970 | 10.4970   | 0.6000             | 5.0000             | 5.0000             | 10.0000            | 16.6667            | 24.0000            |
|        | 2.5   | 23.16        | 8.9625  | 8.8844    |                    |                    |                    |                    |                    |                    |
|        | 5.0   | 20.15        | 7.7988  | 7.7547    |                    |                    |                    |                    |                    |                    |
|        | 7.5   | 17.79        | 6.8875  | 6.8982    |                    |                    |                    |                    |                    |                    |
|        | 10.0  | 15.88        | 6.1439  | 6.2200    |                    |                    |                    |                    |                    |                    |
|        | 20.0  | 13.19        | 5.1038  | _         |                    |                    |                    |                    |                    |                    |
|        | 30.0  | 11.43        | 4.4233  | _         |                    |                    |                    |                    |                    |                    |
|        | 60.0  | 7.91         | 3.0623  | _         |                    |                    |                    |                    |                    |                    |
|        | 120.0 | 7.96         | 3.0816  |           |                    |                    |                    |                    |                    |                    |
|        |       |              |         |           |                    |                    |                    |                    |                    |                    |
| 80     | 0.0   | 27.12        | 10.4970 | 10.4970   | 5.0000             | 5.0000             | 40.0000            | 40.0000            | 0.0083             | 0.0120             |
|        | 5.0   | 6.23         | 2.4127  | 2.4430    |                    |                    |                    |                    |                    |                    |
|        | 10.0  | 5.70         | 2.2050  | 2.1210    |                    |                    |                    |                    |                    |                    |
|        | 15.0  | 5.52         | 2.1375  | 2.0690    |                    |                    |                    |                    |                    |                    |
|        | 20.0  | 5.37         | 2.0775  | 2.0579    |                    |                    |                    |                    |                    |                    |
|        | 25.0  | 5.31         | 2.0550  | 2.0535    |                    |                    |                    |                    |                    |                    |
|        | 30.0  | 5.28         | 2.0454  | 2.0504    |                    |                    |                    |                    |                    |                    |
|        | 60.0  | 3.91         | 1.5118  | _         |                    |                    |                    |                    |                    |                    |
|        | 120.0 | 4.10         | 1.5858  | _         |                    |                    |                    |                    |                    |                    |
|        |       |              |         |           |                    |                    |                    |                    |                    |                    |
| 100    | 0.0   | 27.12        | 10.4970 | 10.4970   | 1.0000             | 1.0000             | 80.0000            | 30.0000            | 16.6667            | 12.0000            |
|        | 1.25  | 5.77         | 2.2345  | 2.2965    |                    |                    |                    |                    |                    |                    |
|        | 2.50  | 3.00         | 1.1625  | 1.0890    |                    |                    |                    |                    |                    |                    |
|        | 3.75  | 2.34         | 0.9075  | 0.8134    |                    |                    |                    |                    |                    |                    |
|        | 5.00  | 1.87         | 0.7240  | 0.7207    |                    |                    |                    |                    |                    |                    |
|        | 6.25  | 1.63         | 0.6300  | 0.6743    |                    |                    |                    |                    |                    |                    |
|        | 7.50  | 1.41         | 0.5475  | 0.6443    |                    |                    |                    |                    |                    |                    |
|        | 8.75  | 1.32         | 0.5100  | 0.6227    |                    |                    |                    |                    |                    |                    |
|        |       |              |         |           |                    |                    |                    |                    |                    |                    |

TABLE II (Continued from previous page.)

| Temp.,<br>°C | Temp,<br>min | Expe<br>[F]<br>% | rimental<br>[F],<br>mole/l. | Predicte<br>[F],<br>mole/l. | $\frac{\underline{k_1}}{\underline{k_1}}$ | $rac{k_2}{k_2^0}$ | $\frac{k_3}{k_3^0}$ | $\frac{k_4}{k_4^0}$ | $rac{k_5}{k_5^0}$ | $rac{k_6}{k_6^0}$ |
|--------------|--------------|------------------|-----------------------------|-----------------------------|---|--------------------|---------------------|---------------------|--------------------|--------------------|
|              | 10.0         | 1.19             | 0.4600                      | 0.6062                      |   |                    |                     |                     |                    |                    |
|              | 20.0         | 1.19             | 0.4600                      | _                           |   |                    |                     |                     |                    |                    |
|              | 30.0         | 1.17             | 0.4524                      |                             |   |                    |                     |                     |                    |                    |
|              | 60.0         | 1.19             | 0.4600                      | —                           |   |                    |                     |                     |                    |                    |
| 120          | 0.0          | 27.12            | 10.4970                     | 10.4970                     | 1.2000                                    | 1.0000             | 150.0000            | 40.0000             | 50.0000            | 12.0000            |
|              | 1.0          | 1.96             | 0.7575                      | 0.7643                      |   |                    |                     |                     |                    |                    |
|              | 2.0          | 1.06             | 0.4125                      | 0.4949                      |   |                    |                     |                     |                    |                    |
|              | 3.0          | 0.86             | 0.3337                      | 0.4312                      |   |                    |                     |                     |                    |                    |
|              | 4.0          | 0.78             | 0.3000                      | 0.3980                      |   |                    |                     |                     |                    |                    |
|              | 5.0          | 0.77             | 0.2977                      | 0.3774                      |   |                    |                     |                     |                    |                    |
|              | 10.0         | 0.47             | 0.1817                      | _                           |   |                    |                     |                     |                    |                    |
|              | 20.0         | 0.32             | 0.1237                      |                             |   |                    |                     |                     |                    |                    |
|              | 30.0         | 0.17             | 0.0657                      | —                           |   |                    |                     |                     |                    |                    |
| 160          | 0.0          | 27.12            | 10.4970                     | 10.4970                     | 0.7000                                    | 1.0000             | 8.0000              | 4.0000              | 1.0833             | 1.5600             |
|              | 1.0          | 3.72             | 1.4412                      | 1.4869                      |   |                    |                     |                     |                    |                    |
|              | 2.0          | 3.29             | 1.2740                      | 1.1963                      |   |                    |                     |                     |                    |                    |
|              | 3.0          | 3.10             | 1.2000                      | 1.1500                      |   |                    |                     |                     |                    |                    |
|              | 4.0          | 2.89             | 1.1175                      | 1.1395                      |   |                    |                     |                     |                    |                    |
|              | 5.0          | 2.77             | 1.0727                      | 1.1369                      |   |                    |                     |                     |                    |                    |
|              | 6.0          | 2.62             | 1.0125                      | 1.1363                      |   |                    |                     |                     |                    |                    |
|              | 10.0         | 2.19             | 0.8492                      | —                           |   |                    |                     |                     |                    |                    |
|              | 20.0         | 1.15             | 0.4470                      |                             |   |                    |                     |                     |                    |                    |
|              | 30.0         | 0.92             | 0.3577                      |                             |   |                    |                     |                     |                    |                    |

TABLE II (Continued from previous page.)

$$k_5^0 = \frac{1.2}{9} k_1^0 \tag{25}$$

$$k_6^0 = \frac{1}{9} k_1^0 \tag{26}$$

Solution of eqs. (18)–(20) produced only a marginal improvement in the comparison of experimental and predicted results at higher temperatures, emphasizing the possible inadequacy of the reaction mechanism proposed. In fact, Sato<sup>13</sup> during a study of the thermodynamics of the urea–formaldehyde reaction concluded that the rate constants remain constant only over narrow conversion limits, because of the change in functionality of urea as the reaction progressed. If, therefore, the rate constants vary with composition as well as temperature, solution of the problem necessitates a simultaneous search for four or six rate constant values at each temperature. The kinetic model then becomes inappropriate, particularly if significant condensation takes place, but it can be used as the basic model to which the experimental data are fitted.

The technique of nonlinear regression analysis was applied<sup>1</sup> to the data to estimate the best values of the rate constants which would predict the experimental data within a specified limit of accuracy. It was assumed that the result



Fig. 5. Sample experimental data; U:F = 1:1.33: (a) 25°C; (b) 40°C; (c) 80°C; (d) 160°C.

at a particular temperature could be applied with acceptable accuracy to a 10-K band around that temperature. The criterion adopted was that the difference between experimentally determined and model predicted values of formaldehyde concentration should be less than the error expected in the chemical determination of free formaldehyde, i.e., 0.1% by weight. Preliminary work with a Honeywell 316 computer indicated that at a U:F ratio of 1:1.33, eqs. (2) and (3) would give satisfactory results for temperatures up to and including 80°C, pro-



Fig. 6. Nelder and Mead logic diagram.

vided that the rate constants were modified. Higher temperatures and all temperatures at a U:F ratio of 1:2.2 required the inclusion of eq. (17).

The optimization exercise was carried out on an ICL 1904S computer using a FORTRAN package<sup>14</sup> which made use of an improved simplex search routine due to Nelder and Mead.<sup>15</sup> The flow sheet is shown in Figure 6. The following inputs were required:

(1) A subroutine for description of the objective function to be minimized:



Fig. 7. Logic diagram for subroutine to define objective function for minimization.

$$S = \sum_{j=1}^{j=n} (E_j - F_j)^2$$

where S = sum squared error function,  $E_j = \text{experimental value of free formal$ dehyde for the*j* $th observation, <math>F_j = \text{predicted value of free formaldehyde for$ the*j*th observation, and <math>n = number of experimental points available. A flow sheet, which also allows for the calculation of predicted formaldehyde concentrations, for the subroutine is shown in Figure 7.

(2) The following variables: number of independent variables; initial starting

values for the rate constants; side length of simplex; reflection coefficient; contraction coefficient; expansion coefficient; and accuracy.

Results are shown in Table II in the form of optimized rate constants relative to those defined in eqs. (13)-(16), (25), and (26). It is apparent that there is no correlation between the optimized rate constants. Therefore each set can only be used to predict formaldehyde concentration variation with time over a narrow temperature range of less than  $\pm 10$  K. Nevertheless, predictions can be made by this method to an acceptable level of accuracy for the early stages of the reaction.

## NOMENCLATURE

| Α                          | constant in Arrhenius eq. (12)                                       |
|----------------------------|--|
| с                          | concentration of reacting specie                                     |
| d                          | differential operator  |
| $E_i$                      | experimental value of free formaldehyde for <i>j</i> th observation  |
| $\Delta E$                 | activation energy  |
| F                          | formaldehyde   |
| $F_i$                      | predicted value of free formaldehyde for <i>j</i> th observation     |
| $k_{1} - k_{6}$            | reaction rate constants, eqs. $(2)$ , $(3)$ and $(17)$               |
| n                          | number of experimental points  |
| r                          | rate of chemical reaction  |
| R                          | universal gas constant   |
| $\boldsymbol{S}$           | sum squared error function   |
| T                          | absolute temperature, K  |
| t                          | time elapsed from start of reaction                                  |
| U                          | urea   |
| $\mathbf{UF}_1$            | monomethylolurea   |
| $UF_2$                     | dimethylolurea   |
| $\overline{\mathrm{UF}_3}$ | trimethylolurea  |
| []                         | concentration of   |
| Superscrip                 | ot   |
| 0                          | applies to rate constant originating from the data of de Jong and de |

Jonge

Subscript

0 initial condition

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